$$\iiint_V (\bar{B}\cdot\nabla\times[\tilde{\phi}]\nabla\times\bar{A} - \bar{A}\cdot\nabla\times[\phi]\nabla\times\bar{B})dv = \oiint_s (\bar{A}\times[\phi]\nabla\times\bar{B} - \bar{B}\times[\tilde{\phi}]\nabla\times\bar{A})\cdot d\bar{S}, \quad (1)$$

where $\bar{A}$ and $\bar{B}$ are vector functions of position, and the surface integral extends over the surface enclosing the volume V. Here $[\phi]$ is a tensor function of position and is not necessarily symmetric. The tilde indicates the transposed tensor. This identity may be derived by applying the divergence theorem to

$$\iiint_V \nabla\cdot(\bar{A}\times[\phi]\nabla\times\bar{B} - \bar{B}\times[\tilde{\phi}]\nabla\times\bar{A})dv, \quad (2)$$

and transforming the volume integral by the identity

$$\nabla\cdot(\bar{F}\times\bar{H}) = \bar{H}\cdot\nabla\times\bar{F} - \bar{F}\cdot\nabla\times\bar{H}. \quad (3)$$

Now let $\bar{B}\equiv\bar{E}$ and $\bar{A}\equiv\bar{G}$, where $\bar{E}$ is the desired electric field intensity due to sources outside of V, and $\bar{G}$ is arbitrary. If $[\phi] \equiv [\mu]^{-1}/j\omega$, (1) becomes

$$\iiint_V (\bar{E}\cdot\nabla\times[\tilde{\mu}]^{-1}\nabla\times\bar{G} - \bar{G}\cdot\nabla\times[\mu]^{-1}\nabla\times\bar{E})dv = \oiint_s (\bar{G}\times[\mu]^{-1}\nabla\times\bar{E} - \bar{E}\times[\tilde{\mu}]^{-1}\nabla\times\bar{G})\cdot d\bar{S}. \quad (4)$$

Now let $\bar{G}$ be a solution of the equation

$$\nabla\times[\tilde{\mu}]^{-1}\nabla\times\bar{G} - \omega^2[\tilde{\epsilon}]\bar{G} = -\bar{u}_p\delta(|\bar{r}-\bar{r}'|), \quad (5)$$

where $\delta(|\bar{r}|)$ is the Dirac delta function and $\bar{r}'$ is within V. The tensors $[\mu]$ and $[\epsilon]$ are, respectively, the tensor permeability and permittivity of the original media in which the fields must be determined, $\bar{u}_p$ is a constant unit vector, and ω is the angular frequency. Time variations of the form $e^{j\omega t}$ are assumed. Since $\bar{G}$ is singular at $\bar{r}=\bar{r}'$, this point must be excluded from V by enclosing it within a small sphere of radius σ as in Fig. 1. In the remaining volume V_1, the volume integrand vanishes identically and the identity of (4) becomes

$$\iint_\sigma (\bar{G}\times[\mu]^{-1}\nabla\times\bar{E} - \bar{E}\times[\tilde{\mu}]^{-1}\nabla\times\bar{G})\cdot d\bar{S} = -\iint_s (\bar{G}\times[\mu]^{-1}\nabla\times\bar{E} - \bar{E}\times[\tilde{\mu}]^{-1}\nabla\times\bar{G})\cdot d\bar{S}. \quad (6)$$

On taking the limit of the left-hand side as $\sigma\rightarrow 0$, and in view of the singularity assumed in (5), one arrives at the result

$$\bar{u}_p\cdot\bar{E}(\bar{r}') = -\oiint_s (\bar{G}\times[\mu]^{-1}\nabla\times\bar{E} - \bar{E}\times[\tilde{\mu}]^{-1}\nabla\times\bar{G})\cdot d\bar{S}, \quad (7)$$

which is the component of $\bar{E}(\bar{r}')$ along the unit vector $\bar{u}_p$. Since $\bar{u}_p$ may be arbitrarily oriented, this amounts to a complete determination of $\bar{E}(\bar{r}')$. This expression is analogous to expressions occurring in the isotropic case with two important differences. First, $[\mu]$ is a tensor, and second, $\bar{G}$ now satisfies a vector wave equation in the transposed media rather than in the original media.

The foregoing was applied to a closed source-free region. It was found that the total field could be expressed in terms of the Green's function and of the fields on the surface bounding the region. Next consider a region with a source as shown in Fig. 2. The technique will be formulated for a point in V', the volume bounded by S and Σ. Call the volume bounded by σ, S and Σ, V_2, and let $\bar{J}^i$ be an impressed electric current density within V_2. Therefore in V' the electric field satisfies

$$\nabla\times[\mu_2]^{-1}\nabla\times\bar{E} - \omega^2[\epsilon_2]\bar{E} = -j\omega\bar{J}^i. \quad (8)$$

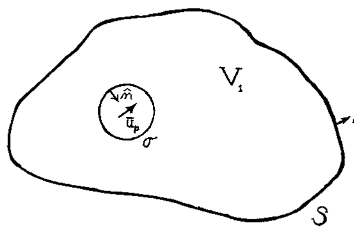

Fig. 1.

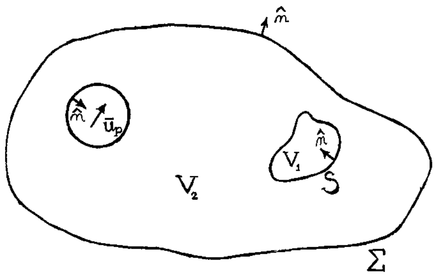

Fig. 2.

With these definitions a procedure similar to that above yields

$$\bar{u}_p\cdot\bar{E}(\bar{r}') = \iiint \bar{G}_2\cdot\bar{J}^i dv - \iint_{\Sigma+s} (\bar{G}_2\times[\mu_2]^{-1}\nabla\times\bar{E} - \bar{E}\times[\tilde{\mu}_2]^{-1}\nabla\times\bar{G}_2)\cdot d\bar{S}. \quad (9)$$

Thus, the field at $\bar{r}'$ in V' is completely determined. If Σ recedes to infinity it may be shown that the integral over this surface vanishes[2] with the result

$$\bar{u}_p\cdot\bar{E}(\bar{r}') = \iiint \bar{G}_2\cdot\bar{J}^i dv - \iint_s (\bar{G}_2\times[\mu_2]^{-1}\nabla\times\bar{E} - \bar{E}\times[\tilde{\mu}_2]^{-1}\nabla\times\bar{G}_2)\cdot d\bar{S}. \quad (10)$$

[2] J. R. Mentzer, "Scattering and Diffraction of Radio Waves," in Pergamon Sci Ser., "Electronics and Waves," Pergamon Press, Inc., New York, N. Y., vol. 7, pp. 12–22; 1955.

Through the modified reciprocity theorem the first term on the right-hand side of (10) becomes

$$\iiint \bar{G}\cdot\bar{J}^i dv = \iiint \bar{u}_p\cdot\bar{E}^i(\bar{r})\delta(|\bar{r}-\bar{r}'|)dv = \bar{u}_p\cdot\bar{E}^i(\bar{r}'). \quad (11)$$

Thus, (10) becomes

$$\bar{u}_p\cdot\bar{E}(\bar{r}') = \bar{u}_p\cdot\bar{E}^i(\bar{r}') - \oiint_s (\bar{G}_2\times[\mu_2]^{-1}\nabla\times\bar{E} - \bar{E}\times[\tilde{\mu}_2]^{-1}\nabla\times\bar{G}_2)\cdot d\bar{S}. \quad (12)$$

This is an example of a modified Green's function technique which may be applied to anisotropic media. The only difference between this technique and that for isotropic media is that the Green's functions used satisfy equations for the transposed media.

Alfred T. Villeneuve
Hughes Aircraft Co.
Culver City, Calif.

N-Way Power Divider*

An N-way power divider is an $(N+1)$ port network with N equal outputs. If we assume the N outputs are symmetrical and the input port (1) is matched, then

$$S = \begin{bmatrix} 0 & S_{12} & S_{12} & S_{12}\cdots S_{12} \\ S_{12} & S_{22} & S_{23} & S_{23}\cdots S_{23} \\ S_{12} & S_{23} & S_{22} & \cdots\cdots S_{23} \\ S_{12} & S_{23} & S_{23} & \\ \cdots & \cdots & \cdots & \cdots S_{23} \\ S_{12} & S_{23} & S_{23} & S_{23}\cdots S_{22} \end{bmatrix}$$

which is a scattering matrix of the $(N+1)$th order. Since the device is lossless, the matrix is unitary. Therefore,

$$N|S_{12}|^2 = 1$$
$$|S_{12}|^2 + |S_{22}|^2 + (N-1)|S_{23}|^2 = 1$$
$$S_{11}S_{12}^* + S_{12}S_{22}^* + (N-1)S_{12}S_{23}^* = 0.$$

The solution to this simultaneous equation indicates that

$$|S_{12}|^2 = \frac{1}{N}$$
$$|S_{23}|^2 = \frac{1}{N^2}$$
$$|S_{22}| = \frac{N-1}{N}.$$

* Received by the PGMTT, December 13, 1960.

It can be concluded that the VSWR into any output port is

$$\underset{(N\neq 1)}{\mathrm{VSWR}_N} = \frac{1+|S_{22}|}{1-|S_{22}|} = 2N-1;$$

the isolation between any two output ports is

$$\mathrm{Isolation}_{(MN)} = 10\log\frac{1}{|S_{23}|^2} = 20\log N$$

$$(M\neq 1)$$
$$(N\neq 1)$$
$$(M\neq N)$$

the coupling between the input and any output port is

$$\underset{(N\neq 1)}{\mathrm{Coupling}_{1N}} = 10\log\frac{1}{|S_{12}|^2} = 10\log N.$$

It can be seen that when multi-outputs are required, say 10 or more, you can obtain some degree of isolation between the output ports. Directional couplers would be required only if there were a necessity for each output port to simulate a matched generator.

HERMAN KAGAN
Bogart Manufacturing Corp.
Brooklyn, N. Y.

Microwave Absorption Modulation by Electron Mobility Variation in n-Type Germanium*

Microwave-radiation attenuation in a dissipative medium such as germanium in which the conductivity and microwave frequency conditions $\sigma < \omega\epsilon$ and $\omega\tau < 1$ are present has been shown by Gibson[1] to be attributed to the absorption constant relation

$$K = 1635\sigma/n \text{ db/meter}, \qquad (1)$$

where τ is the carrier relaxation time, ω the microwave angular frequency, and ϵ the permittivity. In the above equation, σ is the conductivity and n the index of refraction (4.05 in germanium).

Arthur, *et al.*,[2] previously reported that 37.5-kMc radiation absorption in n-type germanium could be modulated by a high external electric field across the crystal. Modulator operation depends on the phenomenon of electron-carrier mobility decreasing as a function of the electric field.[3,4] The semiconductor becomes transparent to the radiation when the carrier slope mobility dv/dE becomes zero in the saturated drift velocity region. Since information is limited on this new type of microwave modulator, a more extensive investigation was made. The present work proposes to investigate the radiation absorption modulation resulting from this effect at a lower frequency and, in addition, to observe the temperature and polarity conditions of the crystal that limit modulator operation.

The experimental arrangement is shown in Fig. 1. Fabrication of the slotted waveguide section through which the sample was inserted consisted of machining a nonradiating slot through each side of the broad section of the rectangular waveguide. The incident 24.2-kMc radiation power was set at 3 mw. An RF substitution method measured the attenuation changes. Measurements of the observed dc signal from a 1N26 crystal detector were made on a Tektronix 545 oscilloscope. The VSWR value with the sample in the microwave field was 1.3, which corresponds to a 0.07-db reflection loss. Polytetrafluoroethylene was placed between the crystal and waveguide section for electrical insulation.

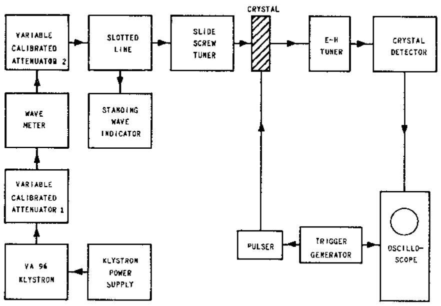

Fig. 1—Block diagram of the experimental apparatus.

Small pieces of 5-ohm-cm n-type germanium were sliced from the bulk single crystal and then lapped to the required thickness. The crystals were highly polished by etching[5] for two minutes in CP4 and rinsed successively in distilled water, ethyl alcohol, and carbon tetrachloride. To reduce hole injection from the positive-going end of each crystal, the wire lead on this end was mounted, using a n-n^+ junction. This junction was made with 95 per cent Sn—5 per cent Sb solder. The solder containing zinc-chloride flux was melted in an argon atmosphere and into this melt, the crystal end was immersed for five minutes at 600°C. This procedure produces alloying of the germanium and the Sn-Sb solder. Ordinary 90 per cent Pb-10 per cent Sn solder was applied for the negative-end connection. The excess flux was removed by placing the crystal in warm distilled water.

The germanium specimens had the form of a rectangular bar whose dimensions were 393 mils long, 125 mils wide, and 5 mils thick. When the crystal was inserted through the waveguide and no voltage pulse was applied, the power attenuated 4.5 db. The crystal was oriented so that the pulsed electric field would be parallel to the RF electric field vector. During operation the crystal was subjected to 0.5-μsec voltage pulses at a repetition rate of 40 pps and fan-cooled. An increase in RF power was detected as the voltage pulse was raised. The change in attenuation as the pulsed electric field across the crystal was increased is shown in Fig. 2. This curve indicates that

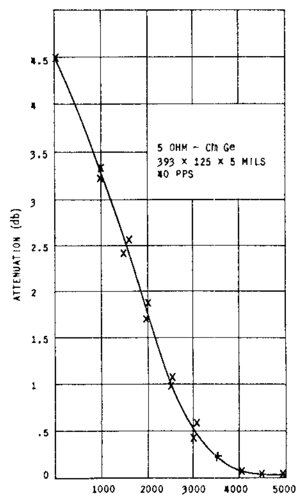

Fig. 2—Absorption variation vs electric field.

practically complete reduction of the 4.5-db attenuation occurs when the electric field across the sample was 4000 v/cm. This field strength represents the onset of the majority-carrier velocity saturation, which is in close agreement with observations made by Arthur.[2] The increase in the RF power when the pulsed electric field across the sample was 4000 v/cm is indicated by the pulse pattern shown in Fig. 3. The amplitude of the RF modulation pulse pattern remained the same when the electric field was increased from 4000 to 5000 v/cm. If the crystal was permitted to operate at a field strength of 3000 v/cm without being cooled, or if the pulse repetition rate was increased to 200 pps and cooling applied, the modulated RF power amplitude was quickly reduced. This microwave absorption effect could be caused by an increase in the carrier density due to thermal ionization. The occurrence, however, was nondestructive. The crystal returned to normal operation when cooling was restored or when the pulse rate of 40 pps was again applied. Fig. 4 indicates the results obtained when the n-n^+ junction was connected initially to the positive-voltage lead and then to the negative-voltage lead when the field strength was 2000 v/cm. From the oscillogram pattern, it is seen that hole injection will limit modulator operation if the n-n^+ junction is not connected to the crystal's positive-going end. It is further seen in Fig. 5 that some crystals with normal operating conditions (cooled and operated at 40 pps) exhibited two regions where RF power modulation changes occurred. These variations occurred

* Received by the PGMTT, October 28, 1960; revised manuscript received, December 23, 1960.

[1] A. F. Gibson, "The absorption of 39-kMc radiation in germanium," *Proc. Phys. Soc. (London) B*, vol. 69, pp. 488–490; March, 1956.

[2] J. B. Arthur, A. F. Gibson, and J. W. Granville, "The effect of high electric fields on the absorption of germanium at microwave frequencies," *J. Electronics*, vol. 2, pp. 145–153; September, 1956.

[3] E. J. Ryder, "Mobility of holes and electrons in high electric fields," *Phys. Rev.*, vol. 90, pp. 766–769; June, 1953.

[4] J. B. Gunn, "The field-dependence of electron mobility in germanium," *J. Electronics*, vol. 2, pp. 87–94; July, 1956.

[5] J. N. Shive, "Intermediate surface treatment," in 'Transistor Technology," Bell Telephone Labs., Inc., vol. 1, pp. 393–406; September, 1952.